OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MATHEMATICS 2642

Probability & Statistics 2

Friday **18 JANUARY 2002** Afternoon 1 hour 20 minutes

Additional materials:
Answer booklet
Graph paper
List of Formulae (MF8)

TIME 1 hour 20 minutes

INSTRUCTIONS TO CANDIDATES

- Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphic calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 60.
- Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
- **You are reminded of the need for clear presentation in your answers.**

This question paper consists of 3 printed pages and 1 blank page.

1 The random variable Y is normally distributed with mean μ and standard deviation 2.50. Given that $P(Y > 12) = 0.3$, find the value of μ. [3]

2 State with justification what approximation, if any, could be made to the distributions of the following random variables, stating the values of any parameters.

(i) A process which generates a random integer between 1 and 100, inclusive, is carried out 75 times. The random variable X is the number of integers generated that are less than or equal to 4. [3]

(ii) $X \sim \text{Po}(20)$. [2]

3 A statistician is collecting data about the sporting interests of the adults in a particular town. She stands at a street corner in that town on a Saturday morning and asks twenty passers-by to complete a questionnaire.

(i) Explain why this method will not produce reliable results. [2]

(ii) Describe a method that will produce more reliable results. [3]

4 At one time on motorways in the UK the average number of potholes was 6 per mile. One randomly chosen mile of the M1 was found to have 9 potholes. Test, at the 5% significance level, whether this evidence suggests that the average number of potholes per mile of the M1 was greater than 6. State your hypotheses clearly, and show the relevant probabilities. [6]

5 A continuous random variable X has the probability density function

$$f(x) = \begin{cases} \dfrac{a}{x^3} & 1 \leqslant x \leqslant 3, \\ 0 & \text{otherwise,} \end{cases}$$

where a is a constant.

(i) Show that $a = \frac{9}{4}$. [3]

(ii) Find $E(X)$. [3]

(iii) Find the value w such that $P(X < w) = 0.3$. [3]

6 A company manufactures electronic resistors. The resistances of the resistors are normally distributed with mean μ ohms and standard deviation 2.4 ohms. A random sample of 16 resistors is used to test the null hypothesis $\mu = 100$ against the alternative hypothesis $\mu \neq 100$ using a 5% significance level. The mean of the sample is denoted by $\bar{x}$.

(i) Determine, correct to 2 decimal places, the range of values of $\bar{x}$ for which the null hypothesis would be rejected. [4]

(ii) Calculate the probability of making a Type II error if the value of μ is in fact 102. [3]

(iii) The company uses four different machines to manufacture the resistors. Three are correctly adjusted, so that they produce resistors with mean 100 ohms, but one is incorrectly adjusted, so that it produces resistors with mean 102 ohms. A sample of 16 resistors is selected at random from the output of one of the machines, chosen at random. Using the rejection region (critical region) found in part **(i)**, calculate the probability that the outcome of the test is to reject the hypothesis $\mu = 100$. [3]

7 At Celtic archaeological sites in a certain region, it is known that on average 30% of gold ornaments are from the Hallstadt culture and the remainder from other cultures. At a newly excavated site 240 gold ornaments were discovered, and of these 88 were from the Hallstadt culture. It may be assumed that these 240 ornaments are a random sample of all gold ornaments at the site. Use a suitable approximation to test, at the 1% significance level, whether this is evidence that more than 30% of all gold ornaments at the newly excavated site are from the Hallstadt culture. State your hypotheses clearly. [11]

8 In a survey of supermarket queues, it is observed that customers arrive at the checkouts independently of one another. The average rate at which customers arrive at the checkouts between 11 a.m. and 3 p.m. is taken to be a constant 3 per minute.

(i) Find the probability that exactly ten customers arrive at the checkouts between 1.00 p.m. and 1.05 p.m. [3]

For the period between 11 a.m. and 3 p.m.,

(ii) use tables to estimate, correct to 2 significant figures, the longest time for which the probability that fewer than 2 customers arrive at the checkouts is greater than 0.06, [3]

(iii) by calculation find the range of values of t for which the probability that no customers arrive in a period of t seconds is greater than 0.000 01. [5]

1 $p(Y \le 12) = 0.7$ $\Phi\left(\frac{12-\mu}{2.5}\right) = 0.7$ $\frac{12-\mu}{2.5} = 0.524$ $\boldsymbol{\mu = 10.7}$ [3]

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2 X~B(75, 0.04) and since $n > 50$ and $np = 3 < 5$ can be approximated by **Po(3)** [3]

Since $\lambda = 20 > 15$ the variable can be approximated as **N(20, 20).** [2]

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3 Her method is unlikely to produce a representative sample because Saturday morning is a prime time for sporting participation and those playing sport (or working) are not available for questioning. [2]

Describe a more reliable method. [3]

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4 $H_0 : \lambda = 6$ $H_1 : \lambda > 6$

On H_0 X~Po(6) $\begin{cases} p(X \ge 9) = 0.1528 \\ p(X \ge 10) = 0.0839 \\ p(X \ge 11) = 0.0426 \end{cases}$ so at 5% level, reject H_0 when $X \ge 11$

In this mile $x = 9$ so insufficient evidence on which to reject H_0. i.e. cannot conclude that there are an above average no. of potholes.

[6]

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5 $1 = \int_1^3 \frac{a}{x^3}\,dx = a\left[-\frac{1}{2x^2}\right]_1^3 = a\left(\frac{1}{2} - \frac{1}{18}\right) = \frac{4}{9}a$ $\therefore \boldsymbol{a = \frac{9}{4}}$

[3]

$E[X] = \int x\,f(x)\,dx = \int_1^3 \frac{a}{x^2}\,dx = \frac{9}{4}\left[-\frac{1}{x}\right]_1^3 = \frac{9}{4}\left(1 - \frac{1}{3}\right) = \boldsymbol{\frac{3}{2}}$

[3]

$p(X < w) = 0.3$ $\frac{9}{4}\left[-\frac{1}{2x^2}\right]_1^w = \frac{3}{10}$ $\frac{1}{2} - \frac{1}{2w^2} = \frac{1}{15}$ $\boldsymbol{w} = \sqrt{\frac{15}{11}} = 1.17$ [3]

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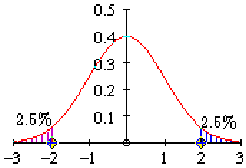
6 $H_0: \ \mu = 100$ $H_1: \ \mu \neq 100$

On H_0 $\dfrac{\bar{X} - 100}{\left(2.4/4\right)} \sim N(0, 1)$

$\dfrac{\bar{X} - 100}{0.6} = \pm 1.96$ $\bar{X} = 98.824, \ 101.176$



critical values are ± 1.96

Reject H_0 if $\boldsymbol{\bar{X} < 98.82}$ **or** $\boldsymbol{\bar{X} > 101.18}$ [4]

$$p\left(\text{Type II error} \middle| \mu = 102\right) = p\left(98.82 < \bar{X} < 101.18 \middle| \mu = 102\right)$$
$$= \Phi\left(\frac{101.176 - 102}{0.6}\right) - \Phi\left(\frac{98.824 - 102}{0.6}\right)$$
$$= \Phi(-1.3733333...) - \Phi(-5.2933333...)$$
$$= \mathbf{0.0848}$$
[3]

$$p\left(\text{reject } H_0\right) = 0.75 \times 0.05 + 0.25 \times (1 - 0.0848)$$
$$= \mathbf{0.266}$$
[3]

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7 H_0: $p = 0.3$ H_1: $p > 0.3$

Working on H_0, the no. of Hallstadt ornaments is $B(240, 0.3) \approx N(72, 50.4)$

For critical value v, ...

$p(< v) = \Phi\left(\dfrac{v - \frac{1}{2} - 72}{\sqrt{50.4}}\right) = 0.9$ $\dfrac{v - \frac{1}{2} - 72}{\sqrt{50.4}} = 2.326$ $v = 89.015..$

Rejection region: "Reject H_0 if $X \geq 90$"

Since $88 < 90$ there is insufficient evidence to conclude that it's a Hallstadt site. [11]

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8 No. of arrivals in 5 min. $X \sim Po(15)$

$$p(X = 10) = e^{-15}\frac{15^{10}}{10!} = \mathbf{0.0486}$$
[3]

$p(< 2) = p(\leq 1) > 0.06 \ \Rightarrow$ $\lambda \leq 4.5$ (from tables)

So longest time period is ${}^{4.5}/_{3} = \mathbf{1.5}$ **minutes**. [3]

$p(0) > 0.00001$

$e^{-t/20} > 0.00001$

$t < -20 \ln 0.00001$

$t < 230.258...$

So the required range is $\mathbf{0 \leq \boldsymbol{t} \leq 230}$ [5]